



## Formation of partial differential equations pdf free printable pdf download

Having done them will, in some cases, significantly reduce the amount of work required in some of the examples we'll be working in this chapter. Fundamental Sets of Solutions - In this section we give a couple of final thoughts on what we will be looking at throughout this course. That in fact was the point of doing some of the examples that we did there. We also examine sketch phase planes/portraits for systems - In this section we will work quick examples illustrating the use of undetermined coefficients and variation of parameters to solve nonhomogeneous systems of differential equations. In addition, we will look at some of the basics of systems of differential equations. In addition, we will define the convolution integral and show how it can be used to take inverse transforms. Separation of Variables - In this section show how the method of Separation of Variables can be applied to a partial differential equations into matrix to solve IVP's that contain Heaviside (or step) functions. We show how to convert a system of differential equations into matrix form. When we do make use of a previous result we will make it very clear where the result is coming from. In addition, we show how to convert an \(n^{ \text{th}}) order differential equations. I've tried to make these notes as self-contained as possible and so all the information needed to read through them is either from a Calculus or Algebra class or contained in other sections of the notes. Laplace Transforms - In this chapter we introduce Laplace transform, what function did we originally have? I try to anticipate as many of the questions as possible when writing these up, but the reality is that I can't anticipate all the questions. With that being said I will, on occasion, work problems off the top of my head when I can to provide more examples than just those in my notes. We do not work a great many examples in this section. The method of undetermined coefficients will work pretty much as it does for nth order differential equations, while variation of parameters will need some extra derivation work to get a formula/process we can use on systems. In general, I try to work problems in class that are different from my notes. Variation of Parameters – In this section we introduce the method of variation of parameters to find particular solutions to nonhomogeneous differential equation. Despite the fact that these are my "class notes", they should be accessible to anyone wanting to learn how to solve differential equations. We define the characteristic polynomial and show how it can be used to find the eigenvalues for a matrix. Here is a listing (and brief description) of the material that is in this set of notes. Solutions to Systems - In this section we will a quick overview on how we solve system and for general external forces to act on the object. This will be one of the few times in this chapter that non-constant coefficient differential equation will be looked at. Included are discussions of using the Ratio Test to determine if a power series, differentiating power series, adding/subtracting power series, i.e. representing a function with a series in the form  $(\sum_{n=1}^{i})$ . We will develop a test that can be used to identify exact differential equations and give a detailed explanation of the solution process. In particular we will look at mixing problems (modeling the amount of a substance dissolved in a liquid and liquid both enters and exits), population problems (modeling a population under a variety of situations in which the population can enter or exit) and falling objects (modeling the velocity of a falling object under the influence of both gravity and air resistance). nonlinear, initial conditions, initial value problem and interval of validity. Heat Equation with Non-Zero Temperature Boundaries - In this section we take a quick look at solving the heat equation in which the boundary conditions are fixed, non-zero temperature. The intent of this chapter is to do nothing more than to give you a feel for the subject and if you'd like to know more taking a class on partial differential equations should probably be your next step. Eigenvalues and Eigenfunctions - In this section we will define eigenvalues and eigenfunctions. The method we'll be taking a look at is that of Separation of Variables. Intervals of Validity - In this section we will give an in depth look at intervals of validity as well as an answer to the existence and uniqueness question for first order differential equations. In addition, we will do a quick review of power series and Taylor series to help with work in the chapter. In addition, we will give a variety of facts about just what a Fourier series will converge to and when we can expect the derivative or integral of a Fourier series to converge to the derivative or integral of the function it represents. Here are my notes for my differential equations course that I teach here at Lamar University. We give an in depth overview of the process used to solve this type of differential equation as well as a derivation of the formula needed for the integrating factor used in the solution process. The Wave Equation - In this section we do a partial derivation of the heat equation that can be solved to give the temperature in a one dimensional bar of length \(L\). In addition, we give solutions to examples for the heat equation, the wave equation and Laplace's equation. We will concentrate mostly on constant coefficient second order differential equations. As we'll see, outside of needing a formula for the Laplace transform of \(y''\), which we can get from the general formula, there is no real difference in how Laplace transforms are used for higher order differential equations. As we will see they are mostly just natural extensions of what we already know who to do. We define the complimentary and particular solution and give the form of the general solution to a nonhomogeneous differential equation. Undetermined Coefficients - In this section we introduce the method of undetermined coefficients to find particular solutions to Differential equation. Series Solutions to Differential equation with a power series. Reduction of Order - In this section we will discuss reduction of order, the process used to derive the solution to the repeated roots case for homogeneous linear second order differential equations, in greater detail. Review : Matrices and Vectors - In this section we will give a brief review of matrices and vectors. As we will see this is exactly the equation we would need to solve if we were looking to find the equilibrium solution (i.e. time independent) for the two dimensional heat equation with no sources. The second topic, Fourier series, is what makes one of the basic solution techniques work. In this section we will use first order differential equations to model physical situations. Here is a brief listing of the topics covered in this chapter. Show Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Included are partial derivations for the Heat Equation and Wave Equation. Euler's Method - In this section we'll take a brief look at a fairly simple method for approximating solutions to differential equations. Due to the mathematics on this site it is best views in landscape mode. We will also give brief overview on using Laplace transforms to solve nonconstant coefficient differential equations. Dirac Delta function - In this section we introduce the Dirac Delta function and derive the Laplace transforms - In this section we ask the opposite question from the previous section. We discuss the table of Laplace transforms used in this material and work a variety of examples illustrating the use of the table of Laplace transforms. We again work a variety of examples illustrating how to use the table of Laplace's equation. Boundary Value Problems & Fourier Series - In this chapter we will introduce two topics that are integral to basic partial differential equations solution methods. Review of some of the basics of power series. Boundary Value Problems - In this section we'll define boundary conditions (as opposed to initial conditions which we should already be familiar with at this point) and the boundary value problem. First Order Differential equations including linear, separable, exact and Bernoulli differential equations. IVP's with Step Functions - In this chapter we will look at several of the standard solution methods for first order Differential equations. IVP's with Step Functions - In this chapter we will look at several of the standard solution methods for first order differential equations. This is the section where the reason for using Laplace transforms really becomes apparent. You should always talk to someone who was in class on the day you missed and compare these notes to their notes and see what the differences are. We also define the Wronskian for systems of differential equations and show how it can be used to determine if we have a general solution to the system of differential equations. We also give a nice relationship between Heaviside and Dirac Delta functions. Repeated Roots - In this section we discuss the solution to homogeneous, linear, second order differential equations, \(ay'' + by' + cy = 0\), in which the roots of the characteristic polynomial, \(ar^{2} + br + c = 0\), are repeated, i.e. double, roots. Modeling - In this section we'll take a quick look at some extensions of some of the modeling we did in previous chapters that lead to systems of differential equations. We also derive the formulas for taking the Laplace transform of functions which involve Heaviside functions. The Heat Equation - In this section we will do a partial derivation of the heat equation that can be solved to give the temperature in a one dimensional bar of length L. Included are derivations for the Taylor series of \({\bf e}^{x}\) and \(\cos(x)\) about \(x = 0\) as well as showing how to write down the Taylor series for a polynomial. We will also take a look at direction fields and how they can be used to determine some of the behavior of solutions. We also show who to construct a series solution for a differential equation about an ordinary point. Partial Differential Equations - In this chapter we introduce Separation of Variables one of the basic solution techniques for solving partial differential equations. Vibrating string - In this section we solve the one dimensional wave equation to get the displacement of a vibrating string. We also take a look at intervals of validity, equilibrium solutions and Euler's Method. We derive the formulas used by Euler's Method and give a brief discussion of the errors in the approximations of the solutions. Higher Order Differential Equations of the previous chapters to differential equations with order higher that 2nd order. Basic Concepts - In this section give an in depth discussion on the process used to solve homogeneous, linear, second order differential equations, \(ay'' + by' + cy = 0\). Note as well that while we example mechanical vibrations in this section a simple change of notation (and corresponding change in what the quantities represent) can move this into almost any other engineering field. Also, I often don't have time in class to work all of the problems in the notes and so you will find that some sections contain problems that weren't worked in class due to time restrictions. Bernoulli Differential equations, i.e. differential equations, i.e. differential equations, i.e. differential equations in the form \(y' + p(t) y = y^{n}\). Terminology - In this section we take a quick look at some of the terminology we will be using in the rest of this chapter. In addition, we also give the two and three dimensional version of the wave equations - In this section we solve separable first order differential equations, i.e. differential equations in the form \(N(y) y' = M(x)\). We will work quite a few examples illustrating how to find eigenvalues and eigenfunctions. Basic Concepts for \(n^{\text{th}}) Order Linear Equations - In this section we'll start the chapter off with a quick look at some of the basic ideas behind solving higher order linear differential equations. Basic Concepts - In this chapter we introduce many of the basic concepts and definitions that are encountered in a typical differential equations course. Review : Systems of Equations - In this section we will also show how to sketch phase portraits associated with complex eigenvalues (centers and spirals). Direction Fields - In this section we discuss direction fields and how to sketch them. We also give a quick reminder of the Principle of Superposition. This will include illustrating how to get a solution that does not involve complex numbers that we usually are after in these cases. Systems of Differential Equations - In this section we'll take a quick look at extending the ideas we discussed for solving \(2 \times 2\) systems of differential equations to systems of size \(3 \times 3\). We will also compute a couple Laplace transforms using the definition. The point of this section is only to illustrate how the method works. Phase Plane - In this section we will give a brief introduction to the phase plane and phase portraits. We will also do a few more interval of validity problems here as well. We will derive the solutions for homogeneous differential equations. We will also define the even extension for a function and work several examples finding the Fourier Cosine Series for a function. We will give a derivation of the solution process to this type of differential equation. We work a wide variety of examples illustrating the initial guess of the form of the particular solution that is needed for the method. We will also work a few examples illustrating some of the interesting differences in using boundary values instead of initial conditions in solving differential equations. We will do this by solving the heat equations. We apply the method to several partial differential equations. Nonconstant Coefficient IVP's - In this section we will give a brief overview of using Laplace transforms to solve some nonconstant coefficient IVP's. Substitutions - In this section we'll pick up where the last section left off and take a look at a couple of other substitutions. We give a detailed examination of the method as well as derive a formula that can be used to find particular solutions. We will also work several examples finding the Fourier Series for a function. The advantage of starting out with this type of differential equation is that the work tends to be not as involved and we can always check our answers if we wish to. Variation of Parameters - In this section we will give a detailed discussion of the process for using variation of parameters for higher order differential equations. In particular we will discuss using solutions to solve differential equations of the form (y' = G(ax + by)). Solving IVPs' with Laplace Transforms - In this section we will examine how to use Laplace transforms to solve IVP's. In addition, we give several possible boundary conditions that can be used in this situation. We only work a couple to illustrate how the process works with Laplace transforms. This is somewhat related to the previous three items, but is important enough to merit its own item. everything in these notes is covered in class and often material or insights not in these notes is covered in class. As we'll most of the process is identical with a few natural extensions to repeated real roots that occur more than twice. We also define the Laplacian in this section and give a version of the heat equation for two or three dimensional situations. Laplace Transforms - In this section we will work a quick example using Laplace transforms to solve a differential equation just to say that we looked at one with order higher than 2nd. We will also derive from the complex roots the standard solution that is typically used in this case that will not involve complex numbers. We define fundamental sets of solutions and discuss how they can be used to get a general solution to a homogeneous Differential Equations - In this section we will discuss the basics of solving nonhomogeneous differential equations. Review : Taylor Series - In this section we give a quick reminder on how to construct the Taylor series for a function. We will also make a couple of quick comments about \(4 \times 4\) systems. Series Solutions - In this section we define ordinary and singular points for a differential equation. We will also give and an alternate method for finding the Wronskian. Systems of Differential Equations - In this chapter we will look at solving systems of differential equations. More on the Wronskian - In this section we will examine how the Wronskian, introduced in the previous section, can be used to determine if two functions are linearly independent or linearly independent. We will also show how to sketch phase portraits associated with real repeated eigenvalues (improper nodes). Fourier Series - In this section we define the Fourier Series, i.e. representing a function with a series in the form  $(\sum_{n \neq x})$ . In one example the best we will be able to do is estimate the eigenvalues as that is something that will happen on a fairly regular basis with these kinds of problems. Linear Equations in the form \(y' + p(t) y = g(t)\). We will restrict ourselves to systems of two linear differential equations for the purposes of the discussion but many of the techniques will extend to larger systems of linear differential equations. We give as wide a variety of Laplace transforms. Fourier Cosine Series - In this section we define the Fourier Cosine Series, i.e. representing a function with a series in the form  $(\sum_{n=0}^{infty} \{A_n\}\cos \left(\frac{n \sin x}{L}\right))$ . Euler Equations – In this section we will discuss how to solve Euler's differential equation,  $(ax^{2}y'' + b x y' + c y = 0)$ . We will also work a couple of examples showing intervals on which  $((\cos \left(\frac{n \sin x}{L}\right)))$  and  $((\sin \left(\frac{n \sin x}{L}\right)))$  and  $((\sin \left(\frac{n \sin x}{L}\right)))$ . \pi x}{L}\right)\) are mutually orthogonal. In this chapter we are going to take a very brief look at one of the more common methods for solving the Heat Equations. Solving the two ordinary differential equations the process generates. We will also show how to sketch phase portraits associated with real distinct eigenvalues (saddle points and nodes). The results of these examples will be very useful for the rest of this chapter and most of the next chapter. be used to take inverse Laplace transforms. Mechanical Vibrations - In this section we will examine mechanical vibrations. THESE NOTES ARE NOT A SUBSTITUTE FOR ATTENDING CLASS!! Using these notes as a substitute for class is liable to get you in trouble. Convergence of Fourier Series - In this section we will define piecewise smooth functions and the periodic extension of a function. We define the equilibrium solution/point for a homogeneous system of differential equations and how phase portraits can be used to determine the stability of the equilibrium solution. Here are a couple of warnings to my students who may be here to get a copy of what happened on a day that you missed. The method illustrated in this section is useful in solving, or at least getting an approximation of the problems are difficult to make up on the spur of the moment and so in this class my class work will follow these notes fairly close as far as worked problems go. We also illustrate its use in solving a differential equation in which the forcing function (i.e. the term without any y's in it) is not known. We also show the formal method of how phase portraits are constructed. Series Solutions - In this section we are going to work a quick example illustrating that the process of finding series solutions for higher order differential equations. We will also define the Wronskian and show how it can be used to determine if a pair of solutions. In addition, we will see that the main difficulty in the higher order cases is simply finding all the roots of the characteristic polynomial. Laplace Transforms - In this section we introduce the way we usually compute Laplace transforms that avoids needing to use the definition. We'll also start looking at finding the interval of validity for the solution to a differential equation. The examples in this section are restricted to differential equations that could be solved without using Laplace transform. The Definition - In this section we give the definition of the Laplace transform. Note that this is in contrast to the previous section when we generally required the boundary conditions to be both fixed and zero. In particular we will define a linear operator, a linear partial differential equation and a homogeneous partial differential equations. Review : Eigenvalues and Eigenvectors of a matrix. This will include deriving a second linearly independent solution that we will need to form the general solution to the system. We derive the characteristic polynomial and discuss how the Principle of Superposition is used to get the general solution. Electronic ISSN 1432-0835 Abstracted and indexed in BFI List Baidu CLOCKSS CNKI CNPIEC Current Contents/Physical, Chemical and Earth Sciences Dimensions EBSCO Discovery Service Google Scholar INSPIRE Japanese Science and Technology Agency (JST) Journal Citation Reports/Science Edition Mathematical Reviews Naver Norwegian Register for Sciencie Edition Mathematical Reviews Naver Norwegian Register for Science Edition Mathematical Reviews Naver Nave ExLibris Summon SCImago SCOPUS Science Citation Index Expanded (SCIE) TD Net Discovery Service UGC-CARE List (India) Wanfang zbMATH Show Mobile Notice Show All Notes Hide make it very clear before we even start this chapter that we are going to be doing nothing more than barely scratching the surface of not only partial differential equations but also of the method of separation of variables. We will look at arithmetic involving matrices and vectors, finding the inverse of a matrix, computing the determinant of a matrix, linearly dependent/independent vectors and converting systems of equations into matrix form. Sometimes a very good question gets asked in class that leads to insights that I've not included here. We do not, however, go any farther in the solution process for the partial differential equations. In a few cases this will simply mean working an example to illustrate that the process doesn't really change, but in most cases there are some issues to discuss. While we do work one of these examples without Laplace transforms, we do it only to show what would be involved if we did try to solve one of the examples without Laplace transforms. quite a bit of work. You will need to find one of your fellow class mates to see if there is something in these notes that wasn't covered in class. Complex Eigenvalues - In this section we will also develop a formula that can be used in these cases. We work a couple of examples of solving differential equations involving Dirac Delta functions and unlike problems with Heaviside functions can be quite involved on occasion. It would take several classes to cover most of the basic techniques for solving partial differential equations. We will also look at how to solve Euler's differential equation. With the introduction of Laplace Transforms we will now be able to solve some Initial Value Problems that we wouldn't be able to solve otherwise. Because I wanted to make this a fairly complete set of notes for anyone wanting to learn differential equations have included some material that I do not usually have time to cover in class and because this changes from semester it is not noted here. Table of Laplace Transforms - This section is the table of Laplace Transforms that we'll be using in the material. Periodic Functions and Orthogonal Functions - In this section we will define periodic functions, orthogonal functions and mutually orthogonal functions and mutually orthogonal functions. We will also define the odd extension for a function and work several examples finding the Fourier Sine Series for a function. Equilibrium solutions, \(y' = f(y)\). Step Functions - In this section we will define equilibrium solutions. (y' = f(y)) have the eigenvalues for a matrix we also show how to find the corresponding eigenvalues for the matrix. We will use reduction of order to derive the second solution in this case. Real Roots - In this section we discuss the solution in this case. Real Roots - In this section we discuss the solution in this case. which the roots of the characteristic polynomial,  $(ar^{2} + br + c = 0)$ , are real distinct roots. Note that while this does not involve a series solution it is included in the series solution to at least one type of differential equation at a singular point. This section will also introduce the idea of using a substitution to help us solve differential equations. Linear Homogeneous Differential equations - In this section we will extend the ideas behind solving 2nd order, linear, homogeneous differential equations to higher order. We will use linear algebra techniques to solve a system of equations as well as give a couple of useful facts about the number of solutions that a system of equations can have. In particular we will model an object connected to a spring and moving up and down. We also investigate how direction fields can be used to determine some information about the solution to a differential equation without actually having the solution. quick example to illustrate that using undetermined coefficients on higher order differential equations is no differential equations with only one small natural extension. We will also convert Laplace's equation to polar coordinates and solve it on a disk of radius \(a\). Summary of Separation of Variables - In this final section we give a quick summary of the method of separation of variables for solving partial differential equations. In addition, we will discuss reduction of order, fundamentals of sets of solutions, Wronskian and mechanical vibrations. Included will be updated definitions/facts for the Principle of Superposition, linearly independent functions and the Wronskian. Also need to discuss how to deal with repeated complex roots, which are now a possibility. We also work a variety of examples showing how to take Laplace transforms that involve Heaviside functions. In particular we will look at mixing problems in which we have two interconnected tanks of water, a predator-prey problem in which populations of both are taken into account and a mechanical vibration problem with two masses, connected with a spring and each connected with a spring and each connected to a wall with a spring. We illustrate how to write a piecewise function in terms of Heaviside functions. Exact Equations - In this section we will discuss identifying and solving exact differential equations. Real Eigenvalues - In this section we will solve systems of two linear differential equations in which the eigenvalues are distinct real numbers. Definitions - In this section we will solve systems of two linear differential equations in which the eigenvalues are distinct real numbers. the equations will run off the side of your device (should be able to scroll to see them) and some of the menu items will be cut off due to the narrow screen width. Repeated Eigenvalues - In this section we will solve systems of two linear differential equations, we give brief discussions on using Laplace transforms to solve systems and some modeling that gives rise to systems of differential equations. Complex Roots - In this section we discuss the solution to homogeneous, linear, second order differential equations, (ay'' + by' + cy = 0), in which the roots of the characteristic polynomial,  $(ar^{2} + br + c = 0)$ , are real distinct roots. Included is an example solving the heat equation on a bar of length \(L\) but instead on a thin circular ring.